

Preface (extract)
& Contents pages (in part)
Only.

Decision-making under uncertainty
- The Bayesian approach

Statistics for Modern Business Decision Making

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Preface

In recent years, Bayesian decision theory has become increasingly popular in applied statistics. Areas in which Bayesian decision theory is applied include quality control, marketing, advertising, and production. More recently, Bayesian theory has been applied in the field of regression analysis, and its concept of a priori information has attracted considerable attention among econometricians.* This book provides a simple and straightforward introduction to Bayesian theory for advanced undergraduates and first-year graduate students in economics and business. The level and arrangement of the text material allow a student with knowledge of high school algebra to understand readily the theory and techniques used.

What This Text Is About. This text covers both classical and Bayesian statistics so that the relationship between these two schools can be clarified. Bayesian decision theory is concerned primarily with the logical analysis of the choice among possible courses of action when the consequences of any course of action depend on the state of nature. While the state of nature is unknown, it is possible to obtain information about it through experiments.

The Bayesian decision theory model explicitly incorporates important assumptions that the classical theory of statistical inference deals with only implicitly. For example, in determining the level of significance or the probability of committing a type I error, a decision maker or a classical statistician considers the relative seriousness of committing type I and type II errors, as well as the a priori probability of occurrence of the various values the state of nature takes. Although these factors are considered in practice, the classical school of statistical inference does not explicitly incorporate them.

* Theil, H., "On Pure and Mixed Estimation in Economics," *International Economic Review*, Vol. 2, No. 1, January 1961; Theil, H., "On the Bayesian Approach to Regression Analysis," Report 6202, Netherland School of Economics, March 1962; Theil, H., "On the Incomplete Prior Information in Regression Analysis," *American Statistical Association Journal*, June 1963; Tiao, George C., and Zellner, Arnold, "Bayes' Theorem and the Use of Prior Knowledge in Regression Analysis," *Biometrika*, Vol. 51, Part 1 and 2, June 1964, pp. 219-230; Tiao, George, and Zellner, Arnold, *Bayesian Analysis of the Regression Model with Autocorrelated Errors*, Report No. 86 (Social Systems Research Institute, The University of Wisconsin, 1964).

This text is divided into four parts. Probability and the classical school of statistical inference are covered in Part I. The methods, characteristics, and limitations of this theory are examined to show why it was necessary to develop Bayesian decision theory. Another advantage of covering this theory before discussing Bayesian theory is that the text can be used as text or reference material for courses in elementary statistics. Topics discussed in Part I include elementary theory of sets, functions, relations, random variables, probability functions, joint probability distributions, and the concepts of expected value and variance. Part I concludes with discussion and evaluation of statistical inference.

Part II deals with Bayesian decision theory (with and without sampling), its relationship to the classical school of statistical inference, and its applications to quality control, production, marketing, and advertising. Arbitrary and binomial probability distributions are assumed. To clarify the meaning of decision theory under conditions of uncertainty, Part II first examines the theory of games and linear programming. The theory of games is considered a special and simple case of decision theory under uncertainty and it exemplifies a large part of the underlying concepts used in decision theory under uncertainty. While linear programming is decision theory under certainty, the discussion will help clarify the similarities and dissimilarities of decision theory under certainty and uncertainty. In addition, the theory of games can be reduced to linear programming and can be solved by the linear programming technique; it also will clarify concepts used in decision theory under uncertainty, such as convex sets, convex combinations, extreme points, and admissible decision rules.

To ensure thorough understanding of Bayesian decision theory, attention is given to analysis in the normal form, which consists of the numeration and evaluation of *all* decision rules, as a means of clarifying the theoretical aspects involved in determining the optimum course of action. This analysis defines the relationship between the decision rules d_i and the classical critical region $c_i \geq a$ where a is a constant.

The principal techniques and concepts discussed in Parts I and II are applied in Part III. A normal distribution is assumed.

Part IV covers the application of Bayesian decision theory to regression analysis, with the primary objective of showing the method of estimating parameters when given a priori information on the state of nature. Both simple and multiple classical regression analysis are covered first to provide background for study of the Bayesian approach to estimating parameters.

Throughout the text, basic abstract models are used to introduce the student to theoretical aspects under consideration. The model is followed by its empirical counterpart relating the theory to real world situations. The theoretical framework is supplemented by numerous examples, and problems and selected reading material are given at the end of each chapter.

The recognized authors in the field of Bayesian decision theory are L. J. Savage, R. Schlaifer, H. Raiffa, H. Chernoff, L. Moses, D. V. Lindley, D. Blackwell, H. Roberts, and L. Weiss. This text owes a considerable debt to

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